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Scaling and four-quark fragmentation

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The conditions for a scaling behaviour from the fragmentation process leading to slow protons are discussed. The scaling referred to implies that the fragmentation functions depend on the light-cone momentum fraction only. It is shown that differences in the fragmentation functions for valence- and sea-quark knock out give rise to scaling violations. It is proposed that these scaling violations are measured experimentally to put constraints on the four-quark fragmentation functions. Estimates suggest that with reasonable assumptions for the fragmentation functions large effects are to be expected.

1. Introduction

In deep inelastic scattering processes on a free proton, generally an intermediate state is produced which decays into a multitude of hadrons. This process is called fragmentation or hadronization. In describing this process we will assume that all energy and momentum involved are transferred to a single quark. To consider a definite case we will use here the example of neutrino scattering on a proton. In this case the hit quark can be either a d-quark (valence or sea) or a ū-quark from the sea. The angle integrated cross-section can be written as

$$\frac{\mathrm{d}\sigma^{\nu p}(x)}{\mathrm{d}x} = \frac{G^2 M_p E}{\pi} \left[x d_v(x) + x d_s(x) + \tfrac{1}{3} x \bar{u}_s(x) \right] , \qquad (1)$$

where $d_v(x)$ ($d_s(x)$) is the valence- (sea-) quark momentum distribution in the proton and $\bar{u}_s(x)$ that of the antiquarks. With the knock out of the valence quark a diquark system with flavor content uu remains. For sea-quark knock out the spectator is a four-quark $\mathrm{uud\bar{d}}$ or uuud system (see fig. 1). Slow protons in the laboratory system, where the target proton is at rest, can originate only from the fragmentation of the spectator system.

Field and Feynman [1] were the first to give a description of the fragmentation process based on scaling arguments in the extreme parton picture, in which the hit quark and the spectator multiquark system are treated independently. Scaling implies here that for hadrons produced with a momentum close to that of one of the partons, the number depends only on a single parameter, z, the light-cone momentum fraction of the hadron with respect to that of the quark system. To describe the observed hadrons one implicitly assumes a factorization of the cross-section into a dynamic part and a fragmentation part and fragmentation functions $D(z)$ are introduced. Thus scaling laws for cross-section ratios for hadron production (see for example ref. [2]) are obtained. The data for the production of energetic hadrons seem to confirm these scaling relations [2,3].

For our purposes the production of low energy hadrons is of interest, for which the basic caveat of the extreme parton model, the fact that energy conservation is violated, becomes an important obstacle. In order to obey energy and momentum conservation the intermediate state is most easily described in terms of a string model [4-6]. In most recent applications of string models [7-9] one typically considers the quarks to be massless and situated at the end of a glue-string. This string can be seen as a convenient parameterization of the strong, non-perturbative color field. The hit quark and the spectator multiquark system are not treated as separate systems but rather as one, connected by a string which has a certain breaking probability. In general the energy of the

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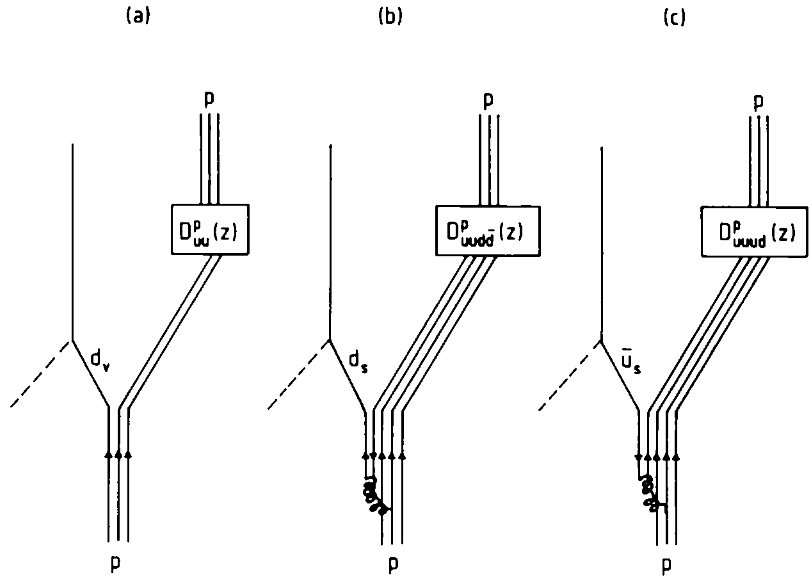

Fig. 1. The three basic processes that give rise to fragmentation in a proton in neutrino scattering. The exchanged virtual boson is denoted by a dashed line.

produced hadrons does not only depend on the light-cone momentum fraction but also on the properties of the string, in particular, its invariant mass. One therefore loses the concept of factorization of the cross-section into a dynamic and a fragmentation part, and thus in general one does not have exact scaling. For the fragmentation of a pure one- or two-quark state, the factorization assumption of Field and Feynman [1] appears to be valid only near the extremes of phase space, i.e. where the hadron has a momentum close to that of the fragmenting quark state. For z close to unity one can therefore still introduce fragmentation functions $D(z)$ depending on the flavor content of the quark system. The kinematic variable z is the light-cone momentum fraction (LCMF) of the slow proton and is equal to the ratio of the light-cone momentum of the proton, $P_p^- = E - p_3$ to that of the string, $P_s^- = M_p + \nu - q$,

$$z = \frac{\sqrt{M_p^2 + p_\perp^2 + p_3^2} - p_3}{M_p + \nu - q} . \qquad (2)$$

The 3-axis is chosen along the direction of the momentum transfer $\boldsymbol{q}$, M_p is the proton mass and all quantities are measured in the laboratory system. The light-cone momentum of the spectator system equals P_s^-, and is therefore the relevant quantity for the fragmentation process leading to a slow particle.

When one considers the production process of fast hadrons in the laboratory system, one deals with the fragmentation process related to the hit quark or antiquark. For this process, even when energy and momentum conservation is included, in most models presently available one basically assumes a factorization of the cross-section and thus obtains a scaling behavior for an appropriately chosen ratio. As remarked, this agrees with the existing data. The main point of the present letter is that this simple full factorization picture generally is no longer valid when one considers hadrons that are slow in the laboratory system, i.e. considers those which originated from he spectator system. Depending on whether a quark or an antiquark is knocked out this spectator can be a two- or a four-quark system. When these hadronize in an identical manner an appropriately chosen ratio of cross-sections is expected to show a scaling behavior (discussed in section 2). Differences in the fragmentation function for the two- and four-quark system reflects itself immediately in a deviation from scaling. In section 3 it is shown that the effect is sizeable for reasonable assumptions for the fragmentation functions. Measuring these scaling violations thus will put sensitive constraints on the (up to now very elusive) four-quark fragmentation function.

2. Scaling

In principle the fragmentation functions for the spectator diquark, triquark–antiquark and the tetraquark systems can differ from each other. In the last part of this letter we will investigate this but temporarily we will assume that they are equal. One can argue that from both the di- and the tetraquark system a $B=1$ object can be formed with the flavor content of the proton through a $q\bar{q}$ pair production in the strong gluon field between the spectator and the hit (anti)quark. For the uu system a $d\bar{d}$ pair creation is necessary with the d joining the 2q system. In the case of the 4q system a $u\bar{u}$ pair should be created with the $\bar{u}$ joining the 4q system. Since, to a good approximation, the u and the d quark have the same mass the two processes will occur with the same probability and thus have the same fragmentation functions,

$$D^{p}_{uu}(z)=D^{p}_{uud\bar{d}}(z)=D^{p}_{uuud}(z)=D^{p}(z)\ . \qquad (3)$$

As discussed in the introduction, for each system as diagrammed in fig. 1 the corresponding contribution to the cross-section approximately factorizes into one term depending on the quark momentum distribution in the target nucleus and a fragmentation function. Using eqs. (1) and (3) the relative cross section for slow protons with a LCMF z can be written as

$$\frac{d\lambda^{\nu p}}{dz}\equiv\frac{d^2\sigma^{\nu p}}{dx\,dz}\bigg/\frac{d\sigma^{\nu p}}{dx}=D^{p}(z)\ . \qquad (4)$$

It is important to notice that only due to assumption of equal fragmentation functions for the two- and four-quark systems the full semi-inclusive cross-section factorizes. Because of this factorization, the RHS of eq. (4) is independent of Bjorken x and the momentum transfer Q^2 and depends only on the single scaling variable z. When plotting $d\lambda^{\nu p}/dz$ versus z all points should thus fall on a single curve, the fragmentation function.

In fig. 2 some current fragmentation functions have been plotted. For the Lund [7] model only the leading order expression [9] is used. The results for the Artru–Mennessier [4] fragmentation description have been obtained from a calculation using the code

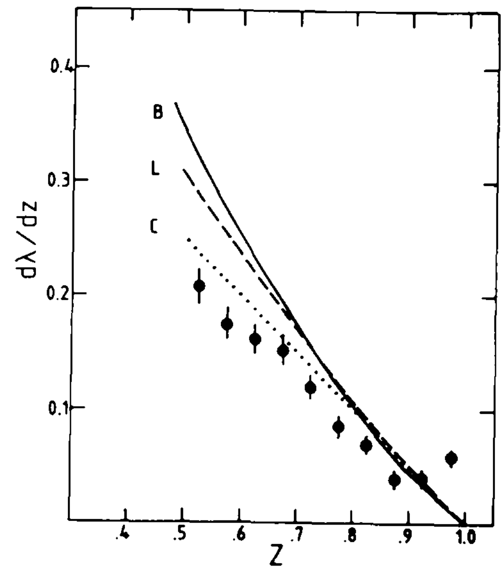

Fig. 2. The fragmentation function $D^{p}_{uu}(z)$ calculated in different models; B: ref. [11], L: ref. [7], C: ref. [10]. The points with error bars are from a calculation using the code VENUS [9], which uses the Artru–Mennessier fragmentation description [4]. In this calculation resonance decay has not been included.

VENUS [9]. In this calculation only d–uu strings with an invariant mass of 6 GeV were considered. These are compared with a linear dependence as is suggested from QCD counting rules [10] and a more refined result using the approach of Field and Feynman [1] due to Bartl [11]. For the interval considered, $0.5<z<1.0$, the differences are only marginal. It should be noted that even though $d\lambda^{\nu p}/dz$ is called a probability distribution it is not normalized to unity, but rather to the average total number of protons that can be produced in the reaction. Furthermore, $z=1$ is an absolute, model independent, kinematic limit. For a free proton at rest it can be shown, using energy and momentum conservation, that only $z<1$ is kinematically allowed.

In eq. (2) z is expressed in terms of the longitudinal momentum p_3 of the observed proton along the direction of $\boldsymbol{q}$. Similarly the probability distribution given in eq. (4) can be rewritten as

$$\frac{d\lambda^{\nu p}}{dz}=\frac{(M_p+\nu-q)E_p}{E_p-p_3}\,\frac{d\lambda^{\nu p}}{dp_3}=z^{-1}\,\frac{d\lambda^{\nu p}}{(dp_3)/E_p}\ . \qquad (5)$$

In this expression the transverse momentum component of the proton only enters in the energy E_p.

3. Breaking of scaling

As mentioned, in principle the di- and tetraquark fragmentation functions are different. Assuming the same fragmentation functions for the two tetraquark systems the probability for a slow proton can be written as

$$\frac{d\lambda^{\nu p}}{dz} = \frac{d_v(x) D^p_{uu}(z) + \frac{4}{3} w(x) D^p_{4q}(z)}{d_v(x) + \frac{4}{3} w(x)} , \tag{6}$$

where $w(x)$ is the sea quark distribution. This expression can be simplified further using the counting rule estimates [10]

$$D^p_{uu}(z) = A(1-z) ,$$
$$D^p_{4q}(z) = D^p_{uudd}(z) = D^p_{uuud}(z) = B(1-z) . \tag{7}$$

The two parameters A and B can in principle be fixed by normalization conditions [11] which also involve the fragmentation functions for meson production which are not considered here. Substituting eq. (7) in eq. (6) one obtains

$$\frac{d\lambda^{\nu p}}{dz} = \left(A + \frac{(B-A) \cdot \frac{4}{3} w(x)}{d_v(x) + \frac{4}{3} w(x)} \right) (1-z) , \tag{8}$$

which, contrary to eq. (4), has an explicit x-dependence (if $A \neq B$). Since meson production off a four-quark jet is expected on the basis of counting rule estimates [10] to be suppressed compared to that of a diquark jet, we will assume $A = 0.5$ and $B = 1.0$. This choice of parameters only serves to show qualitatively what effect can be expected but it is clear that the simple scaling feature is lost; the probability distribution is x-dependent as shown in fig. 3. For large x ($x > 0.5$) the influence of the sea quarks is negligible and one should observe scaling; in this regime the calculated curve is independent of the form of the four-quark fragmentation function. Decreasing the difference between A and B will lessen the effect. As discussed in the next paragraph, in principle even the functional dependence on z can be different for four-quark fragmentation.

As can be seen from fig. 3 the different descriptions of the four-quark fragmentation function give rise to scaling violations, which are as large as 30% at small

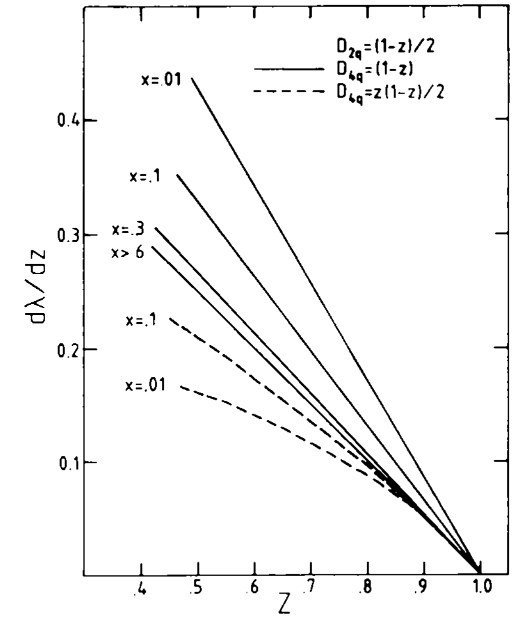

Fig. 3. Magnitudes for scaling violations for two different four-quark fragmentation functions. If the four-quark fragmentation function is chosen equal to that of the two-quark system, the prediction for all Bjorken x values would coincide with the line marked $x > 0.6$. The quark distribution functions have been obtained from a parametrization by Eichten et al. [12].

x. One can distinguish not only a simple scale factor between the two- and four-quark fragmentation function but also a different functional dependence. For instance, fig. 3 also shows the curves obtained from a different fragmentation function,

$$D^p_{4q}(z) = \tfrac{1}{2} z(1-z) , \tag{9}$$

which mimics the fact that in some model descriptions a four-quark cluster preferentially hadronizes near $z = 1$. To distinguish the different models one needs data for the cross-section of proton production which are binned both in x and in z.

For completeness we will mention some other sources for scaling violations. One is the protons coming from the decay of baryonic resonances of which the delta is the most prominent. Since the fragmentation mechanism for forming a delta from a diquark is similar to that for forming a nucleon, this effect can be estimated easily and gives deviations from scaling of less than a few percent. Another possible source of scaling violations might be that a low momentum transfer the invariant mass of the initial string may be small enough that discrete structures in the final state play an important role, i.e. a breakdown of the factorization assumption. We estimate that for invariant string masses exceeding 3 GeV these effects will be of only marginal importance for direct proton production. Since many of the observed protons are due to decay of unstable resonances (the production of which saturates at higher string masses) an invariant mass exceeding 5 GeV is to be preferred.

4. Concluding remarks

In the present letter we have discussed the necessary conditions for scaling to occur in the fragmentation process leading to slow protons in the laboratory system. The essential difference between the presently discussed process and the fragmentation process in which fast hadrons are produced [3,2], is that for slow protons there is a competition between different mechanisms. We showed that this competition can give rise to deviations from the simple scaling and is related to differences in the fragmentation function for diquark and tetraquark jets. By using the concept of a string model the primary effects of energy and momentum conservation (important for $Q^2 < 10\ \mathrm{GeV}^2$) have been included by an appropriate definition of the scaling variable z. It should be noted that this variable, defined in eq. (2), deviates from the definition of z one usually encounters in the literature.

To test the occurrence of scaling in fragmentation, data binned both in x and in the longitudinal proton momentum are necessary. In ref. [13] the number of slow protons in two momentum bins, $150<p<350$ MeV/c and $350<p<600$ MeV/c have been measured as a function of x in a neutrino scattering experiment. Using eq. (2) with $P_s^- = M_p(1-x)$, which is correct in the limit of large Q^2, these data have been replotted in fig. 4. Possible transverse momenta have been ignored in this analysis. The error bars in z quoted in the figure correspond to the integration interval used in ref. [13]. Since no absolute cross-sections are given in ref. [13], we have arbitrarily normalized the data for the low-momentum bin to fall on the same line as those of the high bin. Please note that if another normalization is chosen (20% up for

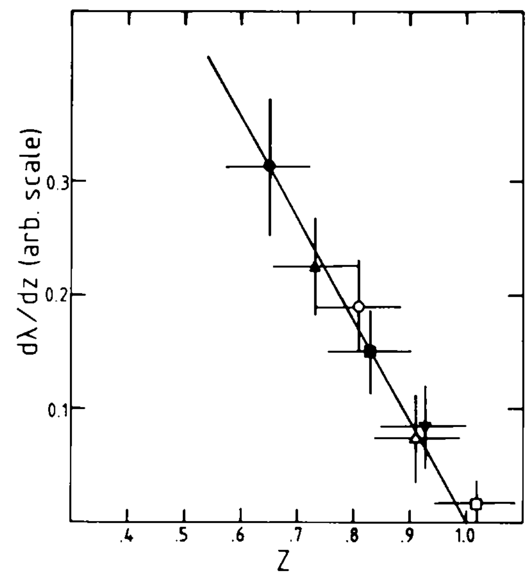

Fig. 4. The fragmentation function as extracted from the data of ref. [13]. The open (solid) points are for the low (high) momentum bin (see text). The circles are for measurements at $x=0.05$, the upward triangles for $x=0.15$, the squares are for $x=0.25$, and the downward triangle is for $x=0.35$. The relative normalization between the high and low momentum bin data has been chosen arbitrarily.

example) there would be definite evidence for a difference between the fragmentation functions. A precise knowledge of the relative normalization for the two momentum bins is thus crucial. Within error bars the data seem consistent with the fragmentation functions shown in fig. 2.

For simplicity we have limited ourselves to neutrino scattering, but the argument can easily be extended to include deep inelastic electron or muon scattering. The advantage of these probes is that precision experiments are feasible. A possible complication is that a strange quark from the sea also can be knocked out. The fragmentation functions for the resulting strange spectator system cannot be assumed to be equal to those of a non-strange spectator. However in general these do not give rise to slow protons.

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